Chapter 5. The massive neutrino theory

1.0. Introduction. Neutrino of Standard Model theory

The present status of the problem of neutrino theory is summarized in (Bilenky, Giunti and Kim, 2000; Glashow, 1961; Weinberg, 1967; Salam, 1969). Namely the theory of electroweak interactions including neutrinos combined with the Quantum Chromo-Dynamics (QCD) is now called the Standard Model (SM).

1.1. Neutrinos features

In the Standard Model neutrinos are strictly massless, $m = 0$; all neutrinos are left-handed, with helicity -1, and all antineutrinos are right-handed, with helicity +1; lepton family number is strictly conserved.

But modern experimental evidence indicates that all of these statements are in fact doubtful (Bilenky, Giunti and Kim, 2000).

1.1.1. Helicity and Chirality

In the neutrino theory the conceptions of helicity and chirality play the important role. In the Standard Model theory, neutrino and antineutrino have opposite helicity. It is mathematically possible that this is in fact the *only* difference between neutrinos and antineutrinos, i.e. a right-handed "neutrino" would be an antineutrino. Particles of this sort are called Majorana particles.

As long as the neutrino is massless, its helicity is completely defined, and a Majorana neutrino would be a different particle from its antineutrino. But if neutrinos have mass, and therefore do not travel at exactly the speed of light, it is possible to define a reference frame in which the helicity would be flipped. This means that there is effectively a mixing between the neutrino and the antineutrino (violating lepton number conservation).

Helicity refers to the relation between a particle's spin and direction of motion. To a particle in motion is associated the axis defined by its momentum, and its helicity is defined by the projection of the particle's spin $\vec{s}$ on this axis: the helicity is the component of angular momentum along the momentum: $h = \dfrac{\vec{s}\vec{p}}{|\vec{s}||\vec{p}|}$.

Thus the helicity operator projects out two physical states, with the spin along or opposite the direction of motion - whether the particle is massive or not. If the spin is projected parallel on the direction of motion, the particle is of right helicity, if the projection is antiparallel to the direction of motion, the particle has left helicity.

Something is *chiral* when it cannot be superimposed on its mirror image, like for example our hands. Like our hands, the chiral objects are also classified into left-chiral and right-chiral objects.

In the Standard Model theory the massless neutrino is described by the Dirac lepton equation without the mass term:

$$\alpha^{\mu}\partial_{\mu}\psi = 0, \quad \mu = 1,2,3,4\,, \tag{1.1}$$

which is also satisfied by $\alpha_5\psi$:

$$\alpha^{\mu}\partial_{\mu}(\alpha_5\psi) = 0\,, \tag{1.2}$$

where the combination of the α matrices, $\alpha_5 = \alpha_0\alpha_1\alpha_2\alpha_3$ has the properties $\alpha_5^2 = 1$ and $\{\alpha_5, \alpha_\mu\} = 0$. This allows us to define the *chirality operators* which project out left-handed and right-handed states:

$$\psi_L = \frac{1}{2}\left(1 - \alpha_5\right)\,\psi \quad \text{and} \quad \psi_R = \frac{1}{2}\left(1 + \alpha_5\right)\,\psi\,, \tag{1.3}$$

where ψ_L and ψ_R satisfy the equations $\alpha_5\psi_L = -\psi_L$ and $\alpha_5\psi_R = \psi_R$, so the chiral fields are eigenfields of α_5, regardless of their mass.

We can express any fermion as $\psi = \psi_L + \psi_R$, so that a massive particle always has a *L*-handed as well as a *R*-handed component. In the massless case ψ however "disintegrates" into separate helicity states: the Dirac equation splits into two independent parts, reformulated as the Weyl equations

$$\frac{\hat{\vec{\sigma}}\hat{\vec{p}}}{\left|\hat{\vec{\sigma}}\right|\left|\hat{\vec{p}}\right|}\left[\frac{1}{2}\left(1 \pm \alpha_5\right)\,\psi\right] = \pm\frac{1}{2}\left(1 \pm \alpha_5\right)\,\psi\,, \tag{1.4}$$

where $\dfrac{\hat{\vec{\sigma}}\hat{\vec{p}}}{\left|\hat{\vec{\sigma}}\right|\left|\hat{\vec{p}}\right|}$ is the helicity operator expressed in terms of the Pauli matrices $\hat{\vec{\sigma}}$.

The **Weyl fermions**, i.e. the massless chiral states $\dfrac{1}{2}\left(1 \pm \alpha_5\right)\,\psi$, are physical since they correspond to eigenstates of the helicity operator. A massless particle, which is in perpetual motion, thus has an unchangeable helicity. The reason is that its momentum cannot be altered, and its spin of course remains unchanged.

1.1.2. Electromagnetic characteristics of neutrino

It is interesting, that, in spite of neutrality, neutrino possesses electromagnetic characteristics. The analysis of these characteristics (Ternov, 2000) helps us to understand the nature of neutrino mass. Electromagnetic properties of Dirac's and Majorano's neutrino appear to be essentially various. Dirac's massive neutrino as a result of the account of interaction with vacuum receives the magnetic moment. And, the neutrino magnetic moment is directed lengthways a spin, and the magnetic moment antineutrino - against a spin. Thus, the particle and the antiparticle differ by the direction of the magnetic moment. For massive Majorano neutrino, identical to its antiparticle, it appears, that it cannot have neither magnetic nor electric moment.

It also appeared, that the mass and the magnetic moment of neutrino are complex nonlinear functions of field strength and energy of a particle.

Moving in an external field, alongside with the magnetic moment, the Dirac neutrino gets as well the dipole electric moment d_ν. Calculations show, that the electric moment of massive Dirac's neutrino, moving in a constant external general view field, is proportional to a pseudo-scalar $\left(\vec{E}\cdot\vec{H}\right)$, which changes sign at the reflection of time. Thus, the electric moment is induced by an external field, if for this field, the pseudo-scalar $\left(\vec{E}\cdot\vec{H}\right)\neq 0$ and its existence does not contradict to T-invariancy of Standard Model. In other words the dipole electric moment of Dirac's neutrino, as well as the magnetic, has dynamic nature.

Note also that there is one electromagnetic characteristic of Dirac's neutrino, which takes place also for Majorano's neutrino: the anapole (or toroidal dipole) moment.

Below we will show that in the framework of the CWED the massive neutrino has the conserved inner (poloidal) helicity, owing to which the above features occur, and is fully described by the Dirac lepton equation.

2.0. Neutrino of CWED

In the previous chapters 2-4 in framework of CWED we have considered the theory of electron and obtained the Dirac electron equation. We have shown that the electron is the twirled ***plane-polarized*** semi-photon EM particle.

Below we will show that the solution of the Dirac equation in bispinor form describes also the motion of the twirled ***circular-polarized*** semi-photon, which is a neutral particle with half spin, but with non-zero mass. We will also show that particle and antiparticle of CWED have opposite inner spiralities. These features and any others compel us to identify this particle with neutrino.

3.0. Plane and circularly polarized electromagnetic waves

Electromagnetic waves emitted by charged particles are in general circularly (or elliptically) polarized (Ivanenko and Sokolov, 1949; Grawford, 1970). Electromagnetic waves are also transverse in the sense that associated electric and magnetic field vectors are both perpendicular to the direction of wave propagation.

Circularly polarized waves carry energy ε and momentum $\vec{p}$ as well as angular momentum $\vec{J}$, which are defined by energy density $U=\frac{1}{8\pi}\left(\vec{E}^2+\vec{H}^2\right)$, momentum density $\vec{g} = \frac{1}{c^2}\vec{S}_P$ and angular momentum flux density, which is given by

$$\vec{s} = \vec{r} \times \vec{g} = \frac{1}{4\pi\ c} r \times \vec{E} \times \vec{H}\ , \tag{3.1}$$

where $\vec{S}_P = \frac{c}{4\pi}\vec{E} \times \vec{H}$ is the Poynting vector indicates not only the magnitude of the energy flux density, but also the direction of energy flow. For simple electromagnetic waves, the Poynting vector is in the same direction as the wave vector. The angular momentum flux density can be checked by the circularly motion of the electron in the circularly polarized wave field (Grawford, 1970).

The figure 1 below shows propagation of electric field associated with a circularly polarized wave with positive (right) and negative (left) helicity.

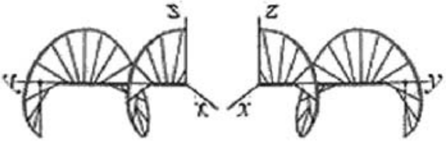

Fig. 1

Positive helicity is the case when the electric field vector is rotated such a right screw would move in the direction of wave propagation (note that in the optics it is called "left hand" circular polarization). Negative helicity (right hand polarization in optics)) refers to rotation in the opposite direction. The direction of the end of the helix indicates the head of the electric field vector, which is rotating around the y axis (as on fig.1).

Since it is impossible by any transformation, except for the spatial reflection, to transfer the right (left) spiral to the left (right) spiral, the *circular polarization of photons is their integral characteristic kept at all transformations, except of the mirror transformation.*

Since the photon helicity is connected to the field rotation, in classical electrodynamics they also talk about rotation of a photon and they enter the photon rotation characteristic – the angular momentum or spin of a photon. In quantum mechanics the spin attributing of a photon has some conditional character. As it is known, as the spin is named the internal angular momentum of a particle in those systems, in which the considered particle is in rest. Therefore in case of a photon, whose speed can not be other than the light speed, it is correct to talk more about the photon helicity than about the spin (Gottfried and Weisskopf, 1984). In this case it is possible to define as helicity the vector (Grawford, 1970).

$$\vec{h}_{ph} \equiv \vec{s}_{ph} = \pm \frac{\varepsilon_{ph}}{\omega} \vec{p}^{\,0} \quad , \tag{3.2}$$

where $\vec{p}^{\,0}$ is the unit Pointing vector, ε_{ph} and ω are the photon energy and circular frequency correspondingly. Apparently the angular momentum value of this vector is equal to $\left|\vec{h}_{ph}\right| = 1\hbar$.

Then according to our hypothesis the helicity vector of neutrino, as of a twirled semi-photon, should have tangential direction to the curvilinear trajectory of twirled wave motion, and its angular momentum should be equal to half of the above value $\left|\vec{h}_{\nu}\right| = \hbar/2$.

4.0. Quantum form of the circularly polarized electromagnetic wave equations

Let us consider the plane electromagnetic wave moving, for example, along y -axis. In general case such wave has two polarizations and contains the following four field vectors: $\vec{\Phi}(y) = \{\mathrm{E}_x, \mathrm{E}_z, \mathrm{H}_x, \mathrm{H}_z\}$

By analogy with the method, used in chapter 2, from wave equation we can obtain two equations in quantum form:

$$\Phi^{+}\left(\hat{\alpha}_o \hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) = 0\,, \tag{4.1'}$$

$$\left(\hat{\alpha}_o \hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right)\Phi = 0\,, \tag{4.1''}$$

Choosing the wave function Φ as (and only as)

$$\Phi = \begin{pmatrix} \mathrm{E}_x \\ \mathrm{E}_z \\ i\mathrm{H}_x \\ i\mathrm{H}_z \end{pmatrix}, \ \Phi^{+} = \begin{pmatrix} \mathrm{E}_x & \mathrm{E}_z & -i\mathrm{H}_x & -i\mathrm{H}_z \end{pmatrix}, \tag{4.2}$$

and putting (4.2) in (4.1) we emerge the following Maxwell equation of advanced and retarded waves :

$$\begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} - \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} - \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} + \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} + \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \quad (4.3) \qquad \begin{cases} \frac{1}{c}\frac{\partial \mathrm{E}_x}{\partial t} + \frac{\partial \mathrm{H}_z}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_z}{\partial t} + \frac{\partial \mathrm{E}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{E}_z}{\partial t} - \frac{\partial \mathrm{H}_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial \mathrm{H}_x}{\partial t} - \frac{\partial \mathrm{E}_z}{\partial y} = 0 \end{cases}, \quad (4.4)$$

The electromagnetic wave equation (and the equations (4.1) and (4.2) also) has the following harmonic solution view (in trigonometric and exponential form correspondingly):

$$\Phi_\mu = A_\mu \sin\left(\omega\, t - \vec{k}\vec{r} + \delta\right), \tag{4.5}$$

$$\Phi_\mu = A_\mu e^{-\frac{i}{\hbar}(\varepsilon t - \vec{p}\vec{r} + \delta)}, \tag{4.6}$$

where $\mu = 1,2,3,4$, A_j are the amplitudes and δ is the constant phase.

Putting here $A_\mu = A_0$, $\delta = 0$, we obtain the following trigonometric form of the equation solutions:

$$\begin{cases} \mathrm{E}_x = A_0 \cos\left(\omega\, t - ky\right) \\ \mathrm{H}_z = -A_0 \cos\left(\omega\, t - ky\right) \\ \mathrm{E}_z = -A_0 \sin\left(\omega\, t - ky\right) \\ \mathrm{H}_x = -A_0 \sin\left(\omega\, t - ky\right) \end{cases}, \quad (4.7') \qquad \begin{cases} \mathrm{E}_x = A_0 \cos\left(\omega\, t - ky\right) \\ \mathrm{H}_z = A_0 \cos\left(\omega\, t - ky\right) \\ \mathrm{E}_z = -A_0 \sin\left(\omega\, t - ky\right) \\ \mathrm{H}_x = A_0 \sin\left(\omega\, t - ky\right) \end{cases}, \quad (4.7'')$$

Let us show that the vectors $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ rotate in the XOZ plane. Actually, putting $y = 0$ we obtain:

$$\vec{\mathrm{E}} = \mathrm{E}_x \vec{i} + \mathrm{E}_z \vec{k} = A_0\left(\vec{i}\cos\omega\, t - \vec{k}\sin\omega\, t\right), \tag{4.8'}$$

$$\vec{\mathrm{H}} = \mathrm{H}_x \vec{i} + \mathrm{H}_z \vec{k} = A_0\left(-\vec{i}\sin\omega\, t - \vec{k}\cos\omega\, t\right), \tag{4.8''}$$

and

$$\vec{\mathrm{E}} = \mathrm{E}_x \vec{i} + \mathrm{E}_z \vec{k} = A_0\left(\vec{i}\cos\omega\, t - \vec{k}\sin\omega\, t\right), \tag{4.9'}$$

$$\vec{\mathrm{H}} = \mathrm{H}_x \vec{i} + \mathrm{H}_z \vec{k} = A_0\left(\vec{i}\sin\omega\, t + \vec{k}\cos\omega\, t\right) \tag{4.9''}$$

where $\vec{i}$, $\vec{k}$ are the unit vectors of the *OX* and *OZ* axes. It is not difficult to show by known algebraic analysis (Jackson, 1999) that we have obtained the cyclic polarised wave. But to keep in evidence we will analyse these relations from geometrical point of view.

Taking into account that the Poynting vector defines the direction of the wave motion:

$$\vec{S}_P = \frac{c}{4\pi}\vec{\mathrm{E}}\times\vec{\mathrm{H}} = -\vec{j}\,\frac{c}{4\pi}\left(\mathrm{E}_x\mathrm{H}_z - \mathrm{E}_z\mathrm{H}_x\right), \tag{4.10}$$

where $\vec{j}$ is the unit vectors of the *OY* axis, and calculating the above (4.10), we have for (4.13') and (4.13''):

$$\vec{S}_P = \frac{c}{4\pi}A_0^2\vec{j}\ , \tag{4.11}$$

$$\vec{S}_P = -\frac{c}{4\pi}A_0^2\vec{j}\,, \tag{4.12}$$

correspondingly. Thus, the photons of the right and left systems (4.8') and (4.9) move in the contrary directions.

Fixing the vector $\vec{\mathrm{E}},\vec{\mathrm{H}}$ positions in two successive time instants ($t_0 = 0$ and $t_1 = t_0 + \Delta t$), we can define the rotation direction. The results are imaged on the figures 2 and 3 correspondingly:

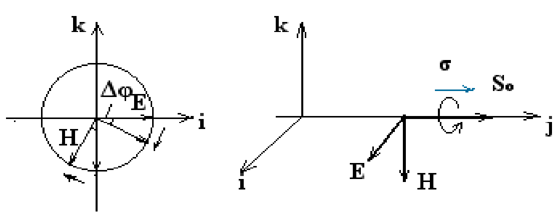

Fig. 2

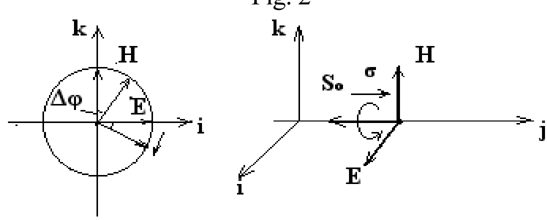

Fig. 3

As we can see the equation sets (4.3 and (4.4) describe the waves with right and left circular polarization correspondingly.

Obviously, by the ring twirling of the circular polarized photon, its helicity does not disappear, but inside the torus become poloidal helicity (or "p-helicity"). At the same time, the movement of fields of a photon along a circular trajectory forms other characteristics of an elementary particle - namely the angular momentum of a particle, or spin. Apparently, the spin of a massive particle and its poloidal angular momentum (p-helicity) are different characteristics. Since these characteristics are the own internal characteristics of a photon, *in the non-linear electromagnetic theory the spin and the poloidal helicity of a particle are independently conserved values.*

We will show now that the Dirac equation with mass term can be considered as the equation of the twirled circularly polarized waves.

5.0. The equation of massive neutrino of CWED

Apparently the massive neutrino must be described by the the Dirac lepton equation with the mass term. Consider the mass term appearing in this case.

Let the circular-polarized wave $\vec{\mathrm{E}},\vec{\mathrm{H}}$, which have the field components $\{\mathrm{E}_x,\mathrm{E}_z,\mathrm{H}_x,\mathrm{H}_z\}$, be twirled with some radius r_{K} in the plane (X',O',Y') of a fixed co-ordinate system (X',Y',Z',O') so that $\mathrm{E}_x,\mathrm{H}_x$ are parallel to the plane (X',O',Y') and $\mathrm{E}_z,\mathrm{H}_z$ are perpendicular to it (see fig. 4)

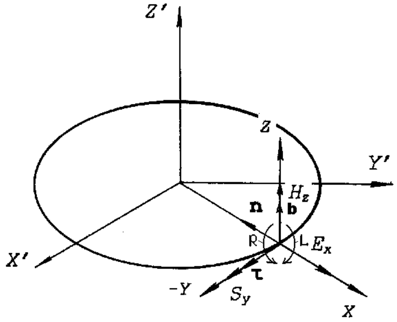

Fig. 4

where the circular arrows shows the right (R) and the left (L) rotation of photon fields.

Let's replace here the unit vectors of the Cartesian coordinate system axes of paragraph 3 $\{\vec{1}_x,\vec{1}_y,\vec{1}_z\}$, connected with a wave, by the vectors of the Frenet-Serret trihedron $\{\vec{n},\vec{\tau},\vec{b}\}$accordingly. Then for the electric and magnetic vectors we have instead of (3.4):

$$\vec{E}(y,t)=\vec{n}E_x+\vec{b}E_z=\left(\vec{n}E_{x0}+\vec{b}E_{z0}\right)e^{i\omega t}, \tag{5.1}$$

$$\vec{H}(y,t)=\vec{n}H_x+\vec{b}H_z=\left(\vec{n}H_{x0}+\vec{b}H_{z0}\right)e^{i\omega t}, \tag{5.2}$$

Here, as well as in the case of the linear polarized EM strings, the unit vector of a normal $\vec{n}$ turns around of $O'Z'$ axis, and bivector $\vec{b}$ remains parallel to it

By analogy to the procedure stated in chapter 2 it is easy to receive the equations of the twirled semi-photons, i.e. the Dirac equation with a mass term.

Unlike to a case of twirling of the plane polarized photons, considered in chapter 2, we do not have basis in advance to approve that here magnetic currents are equal to zero. Really, in first case the magnetic vector was parallel to the rotation axis and kept a constant direction in space, and electric continuously turned around of it, changing direction in space. In the given case the magnetic vector rotates around of a trajectory of motion and is transported along a trajectory just as an electric vector.

Using the procedure stated in chapter 2, we shall show, that in this case there are both electric and magnetic currents.

Let's in the equations of the initial EM strings (4.4) consider the expressions $\vec{j}^e = \frac{1}{4\pi}\frac{\partial \vec{\mathrm{E}}}{\partial t}$ and $\vec{j}^m = \frac{1}{4\pi}\frac{\partial \vec{\mathrm{H}}}{\partial t}$. (Remind that after twirling of EM string, a field vectors of an initial wave $\vec{\mathrm{E}},\vec{\mathrm{H}}$ are transformed in a field vectors of the twirled wave, designated by us in the electromagnetic form as $\vec{E},\vec{H}$, and in the quantum form as ψ). Taking into account that $\frac{\partial \vec{b}}{\partial t} = 0$, we shall receive from (5.1) and (5.2):

$$\frac{\partial \vec{E}}{\partial t} = -\frac{\partial E_x}{\partial t}\vec{n} + \frac{\partial E_z}{\partial t}\vec{b} - E_x\frac{\partial \vec{n}}{\partial t}, \tag{5.3}$$

$$\frac{\partial \vec{H}}{\partial t} = \frac{\partial \vec{H}_x}{\partial t}\vec{n} + \frac{\partial \vec{H}_z}{\partial t}\vec{b} + H_x\frac{\partial \vec{n}}{\partial t}, \tag{5.4}$$

where $\frac{\partial \vec{n}}{\partial t} = -c\kappa\vec{\tau} = -\frac{c}{r_C}\vec{\tau}$, and $r_C = \hbar/mc$. Thus, we receive the electric and magnetic tangential currents, a particularity of which is that they are **alternating**:

$$\vec{j}_\tau^{\,e} = \frac{\omega_p}{4\pi}E_x\cdot\vec{\tau} = \frac{\omega_p}{4\pi}E_{x0}\cdot\vec{\tau}\cdot\cos\omega t\,, \tag{5.5}$$

$$\vec{j}_\tau^{\,m} = -\frac{\omega_{\mathrm{K}}}{4\pi}H_x\cdot\vec{\tau} = -\frac{\omega_{\mathrm{K}}}{4\pi}H_{x0}\cdot\vec{\tau}\cdot\cos\omega t\,, \tag{5.6}$$

Now it is not difficult to see that the quantum form of the equation of circular polarized semi-photons with opposite spirality are the Dirac equations with mass terms.

Taking into account the previous section results, we obtain the twirled semi-photon equations in quantum form, which are equivalent to the Dirac equations:

$$\frac{\partial \psi}{\partial t} - c\hat{\vec{\alpha}}\,\vec{\nabla}\psi - i\hat{\beta}\frac{c}{r_C}\psi = 0, \tag{5.7'}$$

$$\frac{\partial \psi}{\partial t} + c\hat{\vec{\alpha}}\,\vec{\nabla}\psi + i\hat{\beta}\frac{c}{r_C}\psi = 0, \tag{5.7''}$$

and electromagnetic form of these equations:

$$\begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = ij_x^m \end{cases}, (5.8') \quad \begin{cases} \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c}\frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c}\frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = ij_x^m \end{cases}, (5.8'')$$

which are the complex Maxwell equations *with imaginary tangential alternating currents - electric and magnetic*.

We can schematically represent the fields' motion of particles, described by these equations, in the following way (fig. 5):

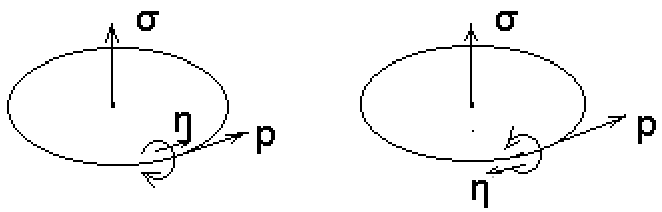

Fig. 5

According to figs. 2 and 3 the semi-photons (fig. 5) have the contrary p-helicities. In the first case the helicity vector and the Poynting vector have the same directions; in the second case they are contrary. Therefore in the non-linear theory we can define the *inner or p-helicity* as the projection of the poloidal rotation momentum on the momentum of the ring field motion.

It is not difficult to show (Davydov, 1965) that actually the helicity is described in CWED by matrix $\hat{\alpha}_5$. Multiplying the Dirac equation on $i\hat{\alpha}_5\hat{\beta}$ and taking in account that $i\hat{\alpha}_5\hat{\beta}\,\vec{\alpha} = \hat{\vec{\sigma}}$, where $\hat{\vec{\sigma}}' = \begin{pmatrix} \hat{\vec{\sigma}} & 0 \\ 0 & \hat{\vec{\sigma}} \end{pmatrix}$ are the spin matrix, and $\hat{\beta}\hat{\alpha}_5 = -\hat{\alpha}_5\hat{\beta},\ \hat{\beta}^2 = 1$, we obtain:

$$\left(i\hat{\beta}\hat{\alpha}_5\hat{\varepsilon} + c\hat{\vec{\sigma}}'\hat{\vec{p}} - imc^2\hat{\alpha}_5\right)\psi = 0, \tag{5.9'}$$

$$\left(i\hat{\beta}\hat{\alpha}_5\hat{\varepsilon} - c\hat{\vec{\sigma}}'\hat{\vec{p}} + imc^2\hat{\alpha}_5\right)\psi = 0, \tag{5.9''}$$

from which we can emerge for the helicity matrix the following expressions:

$$\hat{\alpha}_5 = \frac{c\hat{\vec{\sigma}}'\vec{p}}{i\left(\hat{\beta}\hat{\varepsilon} + mc^2\right)}, \tag{5.10'}$$

$$\hat{\alpha}_5 = \frac{-c\hat{\vec{\sigma}}'\vec{p}}{i\left(\hat{\beta}\hat{\varepsilon} - mc^2\right)}, \tag{5.10''}$$

that connects the $\hat{\alpha}_5$ matrix with helicity, but it is not as in the case $m = 0$.

According to our theory inside the particle the operator $\hat{\alpha}_5$ describes the poloidal rotation of the fields (fig. 5). Remembering that according to the electromagnetic interpretation (see the chapter 3) the value $\psi^+\hat{\alpha}_5\psi$ is the pseudoscalar of electromagnetic theory $\psi^+\hat{\alpha}_5\psi = \vec{E}\cdot\vec{H}$, we can affirm, that in the CWED the p-helicity is the Lorentz-invariant value for the massive particles, and actually it is the origin of the parity non-conservation of the massive particles.

A question arises about how the twirled semi-photon can has a mass and simultaneously doesn't have a charge. It is easy to understand the origin of this difference.

6.0. The charge and mass of EM neutrino

6.1. Demonstration of the neutrino charge absence

The charge is defined by integral on some volume from a current density, which is proportional to the first power of field strength. Obviously, it is possible to have a case when the subintegral expression is not equal to zero, but the integral itself is equal to zero. It is easy to check that we will receive such a result in case when subintegral function changes according to the harmonious law.

It is not difficult to calculate the charge density of the twirled semi-photon particle:

$$\rho_p = \frac{j_\tau}{c} = \frac{1}{4\pi}\frac{\omega_p}{c}E = \frac{1}{4\pi}\frac{1}{r_p}E\,, \tag{6.1}$$

The full charge of the twirled semi-photon can be defined by integrating

$$q = \int_{\Delta\tau_t} \rho_p d\tau\,, \tag{6.2}$$

where $\Delta\tau_t$ is the volume of particle fields.

Using the fig. 4 and taking $\vec{E} = \vec{E}(l)$, where l is the length along the circular trajectory, we obtain:

$$q = \frac{1}{4\pi}\frac{\omega_p}{c}E_o \int \cos k_p l \; d\tau = 0\,, \tag{6.3}$$

(here E_o is the amplitude of the twirled photon wave field, S_c - the area of torus cross-section, ds is the element of the cross-section surface, dl - the element of the length, $k_p = \frac{\omega_p}{c}$ - the wave-vector).

It is easy to understand these results: because the ring current is alternate, the full charge is equal to zero.

6.2. Demonstration of the neutrino mass presence

The particle mass is defined by integral from energy density, which is proportional to the second power of the field strength. In this case the integral is always distinct from zero if the field is distinct from zero.

To calculate the mass we must calculate first the energy density of the electromagnetic field:

$$\rho_\varepsilon = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right), \tag{6.4}$$

In linear approximation (in Gauss's system) we have $\left|\vec{E}\right| = \left|\vec{H}\right|$. Then (6.4) can be written so:

$$\rho_\varepsilon = \frac{1}{4\pi}E^2\,, \tag{6.5}$$

Using (6.5) and a well-known relativistic relationship between a mass and energy densities:

$$\rho_m = \frac{1}{c^2}\rho_\varepsilon\,, \tag{6.6}$$

we obtain:

$$\rho_m = \frac{1}{4\pi\ c^2} E^2 = \frac{1}{4\pi\ c^2} E_o \cos^2 k_s l\ , \qquad (6.7)$$

Using (6.7), we can write for the semi-photon mass:

$$m_s = \int \rho_m d\tau = \frac{E_o^{\ 2}}{\pi\ c^2} \int \cos^2 k_s l\ d\tau \neq 0\ , \qquad (6.8)$$

Obviously this expression never can be equal to zero. Thus actually in the framework of CWED there are the cases when the particle mass do not equal to zero, but the particle charge is equal to zero.

7.0. Topological peculiarities of neutrino-like particle structure

According to our analysis the lepton are the twirled half-periods of photon. In this case neutrino as twirled helicoid represents the Moebius's strip: its field vectors (electric and magnetic) at the end of one coil passes to a state with opposite direction in comparison with the twirled photon vectors, and only by two coils, the vector comes to the starting position (see fig. 6)

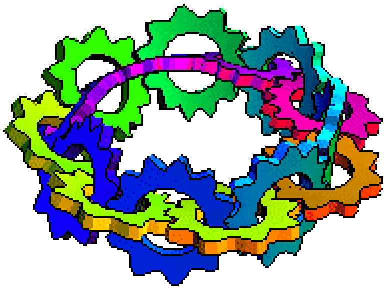

Fig. 6

(fig. 6 is from from (Moebius Strip, MathWord): http://mathworld.wolfram.com/MoebiusStrip.html , where the animation shows a series of gears arranged along a Möbius strip as the electric and magnetic field vectors motion)

Strict verification of the above conclusion about neutrino fields structure follows from the analysis of transformation properties of the twirled semi-photon wave function. (Remember also that a plane electromagnetic wave can be considered as vector combination of two circularly polarized waves rotating in opposite directions).

As it is shown in quantum field theory (Gottfried and Weisskopf, 1984; Ryder, 1985), the rotation matrix possesses a remarkable property (see also section 6 of the chapter 2), wich is illustrated particularly visual by analysis of the neutrino structure.

If the rotation occurs on the angle $\theta = 2\pi$ around any axis (therefore occurs the returning to the initial system of reference) we find, that $U = -1$, instead of $U = 1$ as it was possible to expect. Differently, the state vector of system with spin half, in usual three-dimensional space has ambiguity and passes to itself only after turn to the angle 4π (which accords here to the one wave length of electromagnetic wave).

From above it follows that semi-photon can appear only in CWED, and in classical linear electrodynamics it do not exist.

8.0. Pauli exclusion principle

As we already mention, according to R. Feynman particle, which has the Moebius strip topology, must obey the Pauli exclusion principle. Let show this.

The Pauli exclusion principle can be written in following form: *particles of half-integer spin have antisymmetric wavefunctions, and particles of integer spin have symmetric wavefunctions*.

The answer to the question (Feynman, Leighton, and Sands, 1963), why "particles with half-integral spin are Fermi particles whose amplitudes add with the minus sign", underlies the Fermi statistics, and therefore the Pauli exclusion principle.

There is (Gottfried and Weisskopf, 1986; Gould, 1995) a remarkable property of lepton in three dimensional space: when a lepton is rotated 360 degrees (what means that the wave function phase shifts on 360 degrees), it returns to a state that looks the same geometrically, but that is topologically distinct with respect to its surroundings: a twist has been introduced. A second full rotation (a total of 720 degrees) brings the object back to its original state.

In his last lecture R. Feynman (Feynman, 1987) sketched an elementary argument for above question (see fig 7, which was taken from R. Feynman paper)):

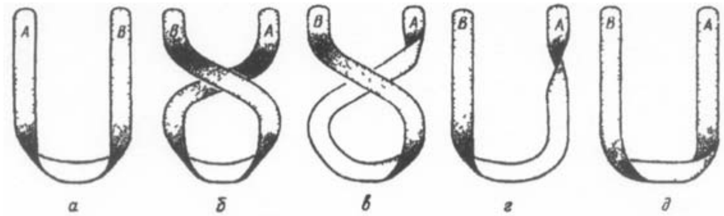

Fig. 7

To see this, first grasp the two ends of a belt, one end in each hand; then interchange the position of your hands. So we have introduced a "twist", which is topologically equivalent to having rotated one end of the belt by 360 degrees.

Thus, when fermions are interchanged, one must keep track of this "implied rotation" and the phase shift, sign change, and destruction interference to which it gives rise. For example, if **A(1)B(2)** describes "electron 1 in state **A** and electron 2 in state **B**," then the state with electrons interchanged must be **-A(2)B(1)** and their superposition is **A(1)B(2) - A(2)B(1)**

Since in the framework of CWED the leptons have the topology of Moebius strip, they must behave as fermions of quantum field theory.

Note also that in CWED two neutrino with left and right poloidal helicity form one twirled circular polarized photon. This corresponds to the theory of Luis de Broglie about the neutrino nature of light (Broglie, 1932a; 1932b; 1934) if we mean the twirled photon, not the "linear" .

9.0. The neutrino of SM and the neutrino of CWED

Let's compare the features of neutrino of CWED with neutrino of SM.

1. it is a lepton, i.e. is described by the Dirac equation;

2. it doesn't have electric charge;

3. particles and antiparticles are distinguished only by helicity;

4. it has all necessary invariant properties according to the theory of the weak interaction.

On the other hand, the CWED neutrino has mass, and the SM neutrino is strictly massless and as such it is described by the Weyl equation. But modern experiments indicate that this statements is doubtful. Thus, a change of SM is necessary, which doesn't violate other advantage of SM.

It was shown that the internal motion of semi-photon fields of neutrino on a circular trajectory is described by the Dirac lepton equation with mass equal to zero, i.e. by the Weil equation. In other words, in framework of CWED the Weyl equation is the equation of internal motion of a neutrino fields as the massless particle (photon fields). As it follows from solution of the Weyl equation and as we have shown above, the internal poloidal rotation (p-helisity) allows the massive neutrino to have properties of the massless neutrino. On the other hand from the outside a "stopped" twirled electromagnetic wave look as the massive neutrino.

If we identify the CWED neutrino with the neutrino of the Standard Model, we eliminate the difficulties of the theory of Standard Model with minimum alteration of the theory. Actually, all that is necessary to overcome the difficulties is to recognize that the neutrino has an internal motion, described by the Weyl equation.